Problem of the week

Thermal energy transfer

A quantity of 0.45 kg of solid chocolate at its melting point is provided thermal energy at a constant rate. The graph shows the variation with time of the temperature of the chocolate.

The specific latent heat of fusion is 1.5×10^5 J kg^{-1}.

(a) Determine the rate at which thermal energy is provided to the chocolate.

(b) Estimate the specific heat capacity of liquid chocolate.

(c) Describe the changes in the internal energy of the chocolate during the melting process.

(d) A piece of this chocolate at its melting point, held in the hand, will melt. Explain why this happens.

(e) Chocolate can be melted by placing the chocolate at its melting point under a heating lamp. Dark and white chocolate have approximately the same specific latent heat of fusion. Equal quantities of dark and white chocolate are placed under the lamp. Suggest which kind of chocolate will completely melt first.

(f) A sphere has mass 3.0 kg, radius 0.36 m, temperature 350 K, specific heat capacity 480 J kg^{-1} K^{-1} and emissivity 0.80. The sphere is radiating into empty space.

(i) Determine the initial rate of change of the sphere's temperature.

(ii) State an assumption made in your estimate.

(g) A rod of length *L* is used to transfer heat from a furnace at a fixed temperature of 500 K to a sink at 300 K.

(i) State three changes to the rod that would result in faster energy transfer.

(ii) estimate the temperature of the rod at a point a distance of 0.8*L* from the left-hand end of the rod.

(iii) State an assumption in the estimate in (ii).

(h) The following data are available for two stars X and Y:

$b_X = 4b_Y$

$R_X = 4R_Y$

$T_X = \frac{T_Y}{2}$

Calculate the ratio $\frac{d_X}{d_Y}$ of distances to the stars.

Answers

(a) Energy provided to melt is $mL = 0.45 \times 1.5 \times 10^5 = 6.75 \times 10^4$ J. Time taken is 5.5 min so the power is $\dfrac{6.75 \times 10^4}{5.5 \times 60} = 204.5 \approx 200$ W.

(b) $mc\Delta T = Pt$ hence $0.45 \times c \times (56 - 32) = 204.5 \times 3.5 \times 60$, giving $c = 4.0 \times 10^3$ J kg^{-1} K^{-1}.

(c) The internal energy is increasing since energy is being provided. The added energy goes into increasing the intermolecular potential energy. None of the energy gets transferred into the random kinetic energy of the molecules since the temperature stays constant.

(d) The hand is at a higher temperature than the chocolate and so heat will flow into the chocolate by the mechanism of conduction.

(e) The dark chocolate absorbs more of the radiant energy from the lamp because it is a better approximation to a black body than the white chocolate. So, it will melt faster.

(f)

(i) $mc\dfrac{\Delta T}{\Delta t} = e\sigma A T^4 \Rightarrow \dfrac{\Delta T}{\Delta t} = \dfrac{e\sigma 4\pi R^2 T^4}{mc}$

$$\frac{\Delta T}{\Delta t} = \frac{0.80 \times 5.67 \times 10^{-8} \times 4\pi \times 0.36^2 \times 350^4}{3.0 \times 480}$$

$$\frac{\Delta T}{\Delta t} = 0.7690 \approx 0.77 \text{ K s}^{-1}$$

(ii) The temperature of the entire sphere is decreasing at the same rate.

(g)

(i) Decrease the length, increase the cross-sectional area, use a rod of higher conductivity.

(ii) $kA\dfrac{(500 - T)}{0.80L} = kA\dfrac{(T - 300)}{0.20L}$ hence, $500 - T = 4(T - 300)$ and so T = 340 K.

(iii) The rod is insulated on its surface.

(h) $b = \dfrac{L}{4\pi d^2} = \dfrac{\sigma 4\pi R^2 T^4}{4\pi d^2} \Rightarrow d = \sqrt{\dfrac{\sigma R^2 T^4}{b}}$. Hence, $\dfrac{d_X}{d_Y} = \sqrt{\dfrac{R_X^2 T_X^4 b_Y}{R_Y^2 T_Y^4 b_X}} = \sqrt{4^2 \times \dfrac{1}{2^4} \times \dfrac{1}{4}} = \dfrac{1}{2}$.